

# **Mathematical Models of Autonomous Logistic Processes**

Scholz-Reiter B, Wirth F, Freitag M, Dashkovskiy S, Jagalski T, de Beer C, Rüffer B

There exist various approaches to the mathematical modelling of dynamic processes occurring in shop floor logistics. These include methods from queuing theory or use dynamical systems given by ordinary or partial differential equations (fluid models). If the number of elements within the process is large it can become prohibitively complex to analyse and optimize a given logistic process or the corresponding mathematical model using global strategies. A new approach is to provide for an autonomy of various smaller entities within the logistic network, i.e. for the possibility of certain elements to make their own decisions. This necessitates changes in the appropriate mathematical models and opens the question of stability of the systems that are designed. In this paper we discuss the fundamental concepts of autonomy within a logistic network and mathematical tools that can be used to model this property. Some remarks concerning the stability properties of the models are made.

## **Introduction**

In a production network (e.g. on shop floor level), the flow of parts is usually pre-planned by a central supervisory or control system. This approach fails for large scale networks in the presence of highly fluctuating demand or unexpected disturbances [21]. One of the reasons for this phenomenon is that in practice the complexity of centralized control architectures tends to grow rapidly with the size of the network, resulting in rapid deterioration of fault tolerance, adaptability and flexibility [25].

An advantageous alternative is the management of the dynamic behaviour according to the requirements of production logistics. In this

sense the development of decentralised and autonomous control strategies is a promising research field [27]. Here autonomous control describes a decentralised coordination of intelligent logistic objects (parts, machines etc.) and the allocation of jobs to machines by the intelligent parts themselves. Therefore, there are no standard policies for production logistics that may be readily applied. Instead, strategic policies have to be derived that enable the parts to decide autonomously, instantaneously and using locally available information only to choose between different alternatives. The application of autonomous control in production networks leads to a coalescence of material flow and information flow and enables every part or product to manage and control its manufacturing process autonomously [7]. The dynamics of such a system depends on the local decision-making processes and produces a system's global behaviour that has new emerging characteristics [20].

In the literature several attempts may be found to explain the emergent behaviour of large scale structures that arise from autonomous control policies. First intuitive approaches suggest to set up a policy like 'go to the machine with the shortest processing time' or 'go to the machine with the lowest buffer level' [28], [29] etc. More sophisticated autonomous control strategies can be found in biological systems. Camazine et al. [11] give a good overview and some case studies of self-organized behaviour in biological systems. Their case studies comprise social insects, slime moulds, bacteria, bark beetles, fireflies and fish. According to the authors biological self-organization can be found in group-level behaviour that arises in most cases from local individual actions that are influenced by the actions of neighbours or predecessors and in structures that are build conjointly by individuals. They identify positive feedback as a "key ingredient" of self-organization. Positive feedback is a method that enables and endorses change in a system. In ant colonies for example, a scout ant that has found food lays down a pheromone trail as it returns to the nest. By changing the environment, succeeding ants may simply follow the trail and find the food, which in turn reinforce the trail with their pheromone [22].

Ant colony optimization (ACO, see e.g. [7], [18]) uses positive feedback with the help of artificial pheromones and is used to solve discrete optimization problems like the travelling salesman problem and the quadratic assignment problem. Logistics applications of the ACO concept can be found for example in Gambardella et al. [19], where the authors find solutions to vehicle routing problems with time windows and in

Bautista et al. [6], where ACO is applied to an assembly line balancing problem for a bike factory. Applications of the pheromone concept for manufacturing control can be found in Peeters et al. [23] and Armbruster et al. [1] where pheromones are used to find a control system for a flexible shop floor.

Brückner et al. [9], [10] suggested implementing the pheromone concept to organize production systems as multi-agent systems. The authors call the approach a “synthetic ecosystems” and present a formal software infrastructure as well as a real-world example. In their “guided manufacturing control system” they combine distributed and reactive control in their control subsystem with a global advisory subsystem.

A concept that uses the interaction between nearest neighbours but does not rely on pheromones is the idea of a bucket brigade, which was introduced by Bartholdi et al. [5]. A bucket brigade is a production line setup, where workers independently follow simple rules that determine what to do next. The rules are: a) Process your work until you meet a downstream worker. If so, give him your work. b) If you do not have work, go upstream until you meet another worker and continue with his job. c) If you are the first worker and you do not have work, then start a new job. d) If you are the last worker, then finish the job and follow rule b). The authors show that such a bucket brigade is self-balancing and results in a global optimum if the workers are sequenced from slowest to fastest. The concept has been extended to bucket brigades with worker learning by Armbruster et al. [2].

In order to develop and analyse autonomous control strategies dynamic models are required. For production systems several model classes have been investigated. These can be divided in discrete and continuous models.

Discrete models are based on the consideration of individual parts in a network of machines. Queuing networks (e.g. with re-entrant lines) can be used to model complex manufacturing systems such as wafer fabrication facilities. The advantage of such models is the possibility to assign decision rules to machines and parts. Stability of such networks is defined probabilistically in terms of Harris recurrence and is often hard to check. For single class networks, which are also called generalized Jackson networks, with work-conserving disciplines such as the FIFO priority discipline or the processor sharing discipline, stability is guaranteed by the

usual traffic condition, which requires that the load is less than the capacity at each machine.

However, this condition is not sufficient for multiclass open queuing networks [12]. Nonetheless, there are fluid limits models that allow the investigation of the stability question for such networks [8], [12]. These are continuous models obtained with help of the functional strong law of large numbers.

A further model class can be derived within the framework of dynamical systems. By time averaging over a representative time period, it is possible to obtain a system of differential equations describing the behaviour of a queuing process as a continuous approximation (see, e.g., [15]). The advantage of this approach is that methods from the theory of dynamical systems can be used. E.g., stability criteria for a class of such systems were recently developed in [15-17]. Continuous models and some stability conditions will be presented later on. Here the term continuous denotes the continuous material flow. In the literature continuous flow models of production systems are often called hybrid models (cf. [4], [12] or [24]), meaning that the material flow is modelled as a continuous flow that is controlled by discrete actions. This discrete control is typical for production systems.

## **Logistic Processes**

Within this paper, we focus on logistic processes on shop floors. Production logistics in this sense encompasses planning, control and monitoring of manufacturing processes. Enterprises face the problem of reacting to dynamically changing market competition in order to deploy and establish high quality products with a reasonable price possibly in a very short time. Thus, production logistics covers the interdisciplinary task between production planning and control, engineering and strategic management. It takes care of the operational control of material and information flows to guarantee efficient and flexible production processes [12].

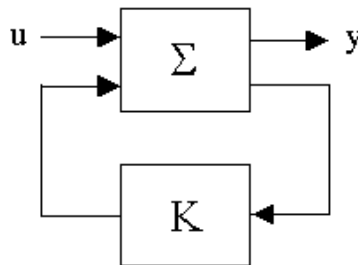
The main goal of production logistics is to design and organise production processes according to high utilisation, low inventory and work-in-process, short throughput times and high adherence to delivery dates. The first two aims are at operational level, whereas the two latter

aims are customer driven. It is obvious that these four aims are mutually contradictory; an enterprise has to find a trade-off between these goals and to position itself according to its own interpretation of their importance.

The main tasks of production logistics can be derived from the main goals. The allocation of orders or jobs to resources comprises of getting (i) the right products or services (ii) at the right time (iii) in the right amount (iv) to the right place. In this section we will discuss how autonomous control can meet these demands in presence of high dynamics.

### Autonomy in Logistic Processes

By autonomy of a logistic process we understand the capability of the process to determine how to react to given changes in the environment, be they fluctuations in demand or in required production rate, failures in some components or changes in the function required of the process. Mathematically speaking we model an autonomous process as an input-output system that is regulated by its own feedback loop with a possibly dynamic feedback, i.e., a feedback capable of using the memory of the system to calculate the control input, see Fig. 1.



**Fig. 1.** A feedback loop

From an abstract point of view it may seem difficult to call a system with inputs autonomous, since in general an input can be used to regulate a system from the outside. The distinction arises through the classification of inputs into inputs directly aimed at low-level control and others. We will call those systems autonomous that receive only inputs in terms of material and information, that needs to be processed, as well as high level demands. The decision on how these high level demands are met using the available resources rests with the control loop of the system. Clearly, the concepts

we are using here are not defined in mathematical terms but would depend on the interpretation of different objects within a concrete scenario<sup>1</sup>.

As an example, consider a two machine two buffer system: Assume that due to customer demand a certain part has to be processed within the system. In the conventional approach a central controlling entity decides based on global information on which buffer-machine system the part is processed. In contrast autonomous control would enable the part to choose the buffer-machine system autonomously based on local information the part actually has access to.

## Mathematical Modelling of Logistic Processes

There are fundamental discrepancies in the interpretation of what constitutes a model depending on different fields of research. In this paper we will take a modest mathematical point of view. We wish to understand the dynamics of logistic processes, that is, the laws by which certain logistic objects or quantities evolve in time. Here logistic objects may be parts in a factory, containers in a transport network or similar things. A model will therefore mostly consist of a set of equations for the time behaviour of a process. These models can be analysed to derive certain global properties of the system or simulated to obtain predictions for specific cases.

The aim of deriving such models is to be able to analyse the behaviour from a qualitative point of view and also to provide predictive models, that is models that are accurate enough to provide good estimates of what is happening in the real process. Based on such a model, control or optimisation strategies may be derived.

Due to the discrete nature of many logistic processes, the earliest models of such processes were in terms of discrete systems with an emphasis on the stochastic nature of the processes, arrival processes and other factors.

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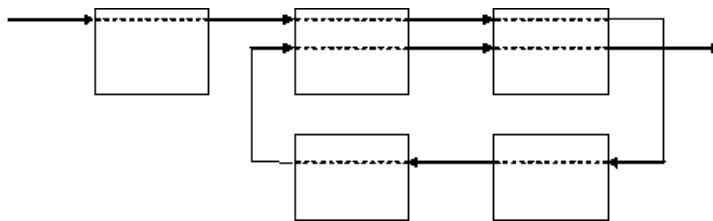
<sup>1</sup> We note that the usage of the word ‘autonomy’ in this paper does not correspond to terminology that is widely used within mathematical systems theory. Here a system is either called autonomous if the laws governing the evolution of the system do not explicitly depend on time [32], or within the framework of behavioural systems, a system is called autonomous, if the behaviours of the system are not parameterised by inputs [26].

We describe such models in the ensuing Section 3.1. In this approach processes are modelled by a number of servers with a processing rate. Each server has one or several queues to which possibly different types of customers arrive. The customers wait in these queues until they are served and after completion of the particular task they go on to the next server or leave the network. Concrete examples where such a modelling approach can be used are job shops where individual machines are interpreted as servers and customers are the parts that have to be processed. In the later sections we present continuous models in which parts and also production stage are not modelled as discrete variables.

### Discrete Models and Fluid Approximations

Let  $J$  be the number of single machines denoted by index  $i=1,\dots,J$ . There are  $K$  classes of parts being processed. Each class  $k=1,\dots,K$  has its own exogenous arrival process with interarrival times  $t_k(n)$ ,  $n=1,2,\dots$  with  $t_k(n)=\infty$  for all  $n$  for some class  $k$  meaning that there are no external arrivals for this class.

Parts of class  $k$  require service at machine  $s(k)$  and their service times are  $T_k(n)$ ,  $n=1,2,\dots$ . After being processed at station  $s(k)$  a class  $k$  part becomes a part of class  $l$  with probability  $P_{kl}$  or exits the network with probability  $1-\sum_l P_{kl}$ , independent of all previous history, where  $P=(P_{kl})$  is a substochastic matrix which is called routing matrix. Such a network is called an open multiclass queuing network, or briefly multiclass network. In case there is only one class with exogenous arrivals and the entries of the routing matrix satisfy  $P_{k,k+1}=1$ , for  $k=1,\dots,K-1$  and zero otherwise, then the multiclass network is called a re-entrant line, see Fig. 2.



**Fig. 2.** A seven buffer five machine re-entrant line

Such models have been considered by many authors, see e.g. [14]. The fluid limit models for multiclass networks and re-entrant lines were considered by, e.g., [13], [14], where the stability question is discussed and stability criteria via fluid models are obtained.

Within this modelling framework autonomous control can be introduced as follows. If the transition probabilities  $P_{kl}$  are dependent on the current buffer level of classes, this dependence can reflect the ability of parts to decide where to go to. Furthermore, the distribution of  $T_k$  can also depend on the state of the queues; this reflects the ability of machines to change their own processing rate. Finally, servers may be able to decide in which order to process the waiting parts on the base of their buffer levels, i.e., the serving discipline is changing with time. Stability investigation and fluid models have yet to be developed for such re-entrant lines with autonomous control.

### Continuous Models: Partial Differential Equations

We now describe a modelling approach based on partial differential equations. We introduce the variable  $x$  taking values in  $[0,1]$  which signifies the completion stage within a certain production process, see [4]. So material at the stage  $x=0$  stands for raw material, while the material has reached stage  $x=1$  when production process is completed. In this approach we are interested in the density function  $\rho(x,t)$  which denotes the amount of material that has reached completion stage  $x$  at time  $t$ . The approach is now to write down a partial differential equation for  $\rho$ . The first of the following equations represents conservation of mass, while the second is an equation for the local velocity within the production system, cf. [3].

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho v(\rho))}{\partial x} = 0, \quad x \in (0,1),$$

$$v(\rho(x,t)) = v_0 \left( 1 - \frac{\rho(x,t)}{\rho_m} \right).$$

The advantages of this modelling approach lie in the relative ease with which model based simulations can be performed. For logistic processes with a large number of production stages it is also plausible to justify the transition from a finite number of production stages to a continuum. However, the approach does not lend itself easily to the modelling of autonomy because it is not obvious how to incorporate the behaviour of autonomous parts in the PDE. For instance one of the problems occurring is that for autonomous parts there may not be an ordered set of stages that



has to be completed, so that it does not really seem appropriate to model completion by a variable taking values in  $[0,1]$ . While this does not mean that the approach is not suitable for modelling autonomous processes, the derivation of such models is an open problem.

### Continuous Models: Ordinary Differential Equations

In this section we first consider a single autonomous machine that can be modelled in a continuous modelling framework. Then we will show how such machines can be combined in a logistic network.

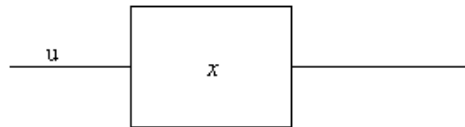
#### A Single Machine

Let  $x=(x^1,\dots,x^n)$  be the vector representing the state of a machine at time  $t$  and let  $u=(u^1,\dots,u^k)$  be the vector of inputs representing both external disturbances and inputs from other machines, see Fig. 3. The evolution of the state  $x$  with time  $t$  is described by a differential equation

$$\frac{dx}{dt} = f(x,u)$$

with initial condition

$$x(0) = x^0$$



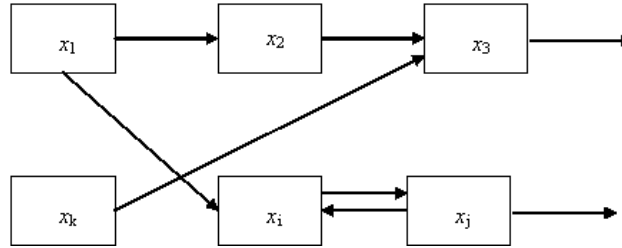
**Fig. 3.** A single machine

The decision rules of the machine are included in the function  $f$ . The input  $u$  accounts also for the decisions of the processed parts. Stability properties of such a nonlinear system can be described in terms of input-to-state stability (ISS), see [31].

#### A Production Network

Consider a shop floor with several, that is  $m$  machines. To each of these we associate its state vector denoted by  $x_i=(x^1,\dots,x^n)\in R^n$ ,  $i=1\dots,m$ , and denote the total state of the network by  $x=(x_1,\dots,x_m)\in R^{mn}$ . Let us combine these machines in a network, see Fig. 4. This network may be represented

as a directed graph, where the nodes are individual machines and edges describe an influence of the state of one machine on the state of another machine.



**Fig. 4.** A network of machines with mutual influences represented as a directed graph

The notion of ISS incorporates a measure of influence of the magnitude of the input to the magnitude of state, called nonlinear gain. A nonlinear gain  $\gamma_{ij}$  from machine  $x_i$  to machine  $x_j$  is a strictly increasing continuous function with  $\gamma_{ij}(0)=0$  [31]. These gains can be gathered into a matrix, setting  $\gamma_{ii} \equiv 0$ , which is a weighted adjacency matrix of the graph representation of the production network. Based on this a stability condition can be derived.

The dynamical behaviour of this network is given by a system of differential equations

$$\frac{dx_1}{dt} = f_1(x_1, \dots, x_m, u)$$

$$\frac{dx_m}{dt} = f_m(x_1, \dots, x_m, u)$$

with initial conditions

$$x_1(0) = x_1^0, \dots, x_m(0) = x_m^0.$$

### Modelling Autonomy in Logistic Processes

As we have seen in the brief discussions of the previous sections it is not obvious how to include the concept of autonomy in the mathematical

models, depending on the modelling approach. In general existing models aim for a global understanding of the system and are suitable for the derivation of global control strategies. The implementation of such strategies may be unfeasible due to the size of the network, problems in making information available globally within a network and the like. This is the intrinsic motivation for studying autonomous control processes. Autonomy of processes suggests to model each process individually and to derive a model for the overall systems by coupling the autonomous components. Such an approach has been studied in the area of decentralised control, which we will now briefly discuss.

In the field of control theory decentralised control has been actively investigated starting in the early 80s of the last century, see [30, 33] for an account and an introduction to the available results. The basic paradigm of decentralised control is that in contrast to the situation depicted in Fig. 1, a system is to be controlled by several controllers each of which only has access to a subset of the measured variables and to the control inputs to perform its task. This raises the question under which conditions a global control goal can be reached via the implementation of several local controllers. Especially for linear systems several results have been obtained that characterise stabilisability and optimisation of systems in which only an approach using decentralised strategies is possible, see [30, 33]. For nonlinear systems however, many basic questions remain unsolved.

From a certain point of view the problem of designing logistic processes with several autonomous components can be viewed as a variant of the problems treated in the field of decentralised control. Also in the logistic context the goal is to achieve certain tasks by the actions of several independent processes, each of which has limited access to the information. One of the fundamental difficulties in this approach is that very often logistic processes are governed by nonlinear laws. In other cases, one wishes to introduce nonlinearities to achieve certain control goals. In this area many mathematical problems are still unsolved.

## Autonomous Control and its Effects on the Dynamics of Logistic Processes

Here we give some examples, how autonomous control can be introduced into the models discussed above and we consider how it affects the solutions of these models.

First consider the re-entrant line discussed above. As we have noted there, the possibility to choose where to go to be served for the parts can be described in terms of the transition probabilities  $P_{ij}$ , making them dependent on the current situation, e.g., on the queue lengths. From the other side, if the machines are able to increase their processing rate when their queues are long or to decrease it once the queues become short, the service times  $T_k(n)$  become also functions of the queue lengths. Appropriately chosen rules of the autonomous control may improve the dynamics of the production line in the sense that it becomes more efficient and robust. The resources of idling machines can be utilised. The parts automatically go to an idling machine, i.e., one with an empty queue, if the others are busy, i.e., have longer queues. In case of failure of a machine the parts route themselves to other machines. The ability to change service rates may help to avoid bottlenecks. These are potential advantages of an autonomous control. However the rules of an autonomous control should be chosen carefully. There are examples (see [8]) of networks satisfying the usual traffic condition that the nominal load of the whole network is less than one, but that are nonetheless unstable, i.e., the queues grow unboundedly.

### An Illustrative Example

Let us consider a couple of simple deterministic scenarios to demonstrate what a continuous model looks like in case of autonomous control. We consider a two machine production network. In this network there are two types of parts arriving at rates  $a_i$ ,  $i=1,2$ , to receive service at the two different machines. The first machine is designed to process the first type of parts at rate  $b_{11}$ , however, it is able to process parts of the second type at a reduced rate  $b_{12} < b_{11}$ . Similarly, for the second machine we have the two processing rates  $b_{22} > b_{21}$ , for serving the second and the first type, respectively. If there is no control of the particle routing, parts of each type

are always served at the machine designed for their type, i.e., a part of type  $i$  goes always to the  $i$ -th machine. This situation we will call Scenario 1.

In the second scenario the parts are able to decide by themselves at what machine they want to be serviced. They use certain decision rules that form the autonomous control and that have to be defined in advance. For example, a part might choose the machine with the shortest queue. Here we will use the following decision rule: A part of type  $i$  is routed to the machine  $j \neq i$  only if the queue in front of machine  $j$  is empty and at the same time the queue in front of machine  $i$  is positive. Otherwise, it chooses the machine  $i$ . In case of  $a_i > b_{ii}$ ,  $i=1,2$ , both queues eventually become positive and each part of type  $i$  goes to the  $i$ -th machine. This case is not interesting for us. The situation is similar if  $a_i < b_{ii}$ ,  $i=1,2$ . An interesting setup is  $a_1 < b_{11}$ , and  $a_2 > b_{22}$ . In this case the first machine, which would idle periodically in the first scenario, every now and then receives parts of the second type. Hence the total throughput should be not less than in the first scenario.

We consider also the following Scenario 3, but with a different autonomous control. The parts first arrive at a common buffer. Then, when the  $i$ -th machine completes service, it orders a part of type  $i$  from the buffer. If there are no parts of type  $i$ , it orders a part of the other type. One can say that in this scenario the machines are autonomously controlled. The machines decide which type of part to process next. One way to compare these three scenarios is via discrete event simulation, which we do before we turn to continuous models.

### Discrete-Event Simulation

It is clear that the interesting case is  $a_1 < b_{11}$  and  $a_2 > b_{22}$ . To perform the simulation we normalise the maximum arrival rate of the parts of the second type to be one and set  $a_1 = 1/24$ ,  $b_{11} = b_{22} = 1/16$ ,  $b_{12} = b_{21} = 1/20$ . The arrival rate of the second type is varied between  $1/16 < a_2 < 1$ . The simulation result of a time period of 500 time units is presented in Figure 5, where the total amount of parts processed by both servers is plotted. Dashed, solid, and dotted lines correspond to Scenarios 1, 2, and 3, respectively. In the first scenario there are no decision rules, and hence the total throughput depends only on the processing rates, but not on the arrival rates. The second scenario is more efficient than the first one for most choices of arrival rate  $a_2$ . As expected, the third scenario has an even higher throughput than the first two. For longer interarrival times  $1/a_2$  of

parts of the second type all three graphs coincide. This is clear, since in this case the second machine can serve all arriving parts of the second type.

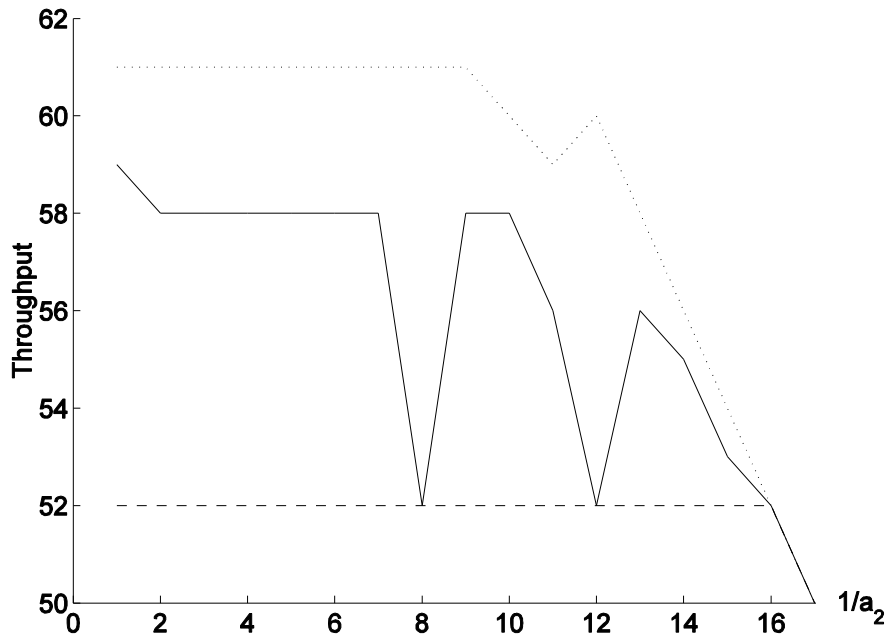


Fig. 5. Total throughput depending on the arrival rate of parts of second type

### The Continuous Model

Let  $x_i(t)$ ,  $y_i(t)$  denote the number of parts of the first and second type, respectively, waiting in buffer  $i$ . Denote by  $0 \leq p_i(t) \leq 1$  the fraction of parts of type  $i$  that are routed to machine 1 at time  $t$ . The evolution of these state variables can be described by ordinary differential equations as

$$\begin{aligned}
 x_1(t) &= p_1(t)a_1(t) - b_1(t) \frac{x_1(t)}{x_1(t) + y_1(t)} \\
 y_1(t) &= p_2(t)a_2(t) - b_1(t) \frac{x_1(t)}{x_1(t) + y_1(t)} \\
 x_2(t) &= (1 - p_2(t))a_2(t) - b_2(t) \frac{x_2(t)}{x_2(t) + y_2(t)} \\
 y_2(t) &= (1 - p_1(t))a_1(t) - b_2(t) \frac{x_2(t)}{x_2(t) + y_2(t)}
 \end{aligned}$$

see [15]. The processing times of the machines are not constant but depend on the mixture of served parts, i.e., their fractions, which may change over time due to autonomous control of the parts. Moreover the processing rates are discontinuous functions of time and their expressions depend on the situation at the queues. If both queues are nonempty then

$$b_i(t) = \frac{\frac{x_i(t)}{b_{ii}} + \frac{y_i(t)}{b_{ij}}}{\frac{x_i(t)}{b_{ii}} + \frac{y_i(t)}{b_{ij}}}, \quad i = 1, 2,$$

(see [15] for details). If the first buffer is empty,  $x_1(t)+y_1(t)=0$ , i.e.,  $x_1(t)=y_1(t)=0$ , it holds that

$$\begin{aligned}
 b_1(t) &= \min \left( p_1(t)a_1(t) + p_2(t)a_2(t), \frac{p_1(t)a_1(t) + p_2(t)a_2(t)}{\frac{p_1(t)a_1(t)}{b_{11}} + \frac{p_2(t)a_2(t)}{b_{12}}} \right) \\
 b_2(t) &= \min \left( (1 - p_1(t))a_1(t) + (1 - p_2(t))a_2(t), \frac{(1 - p_1(t))a_1(t) + (1 - p_2(t))a_2(t)}{\frac{(1 - p_1(t))a_1(t)}{b_{11}} + \frac{(1 - p_2(t))a_2(t)}{b_{12}}} \right)
 \end{aligned}$$

The rules of autonomous control are encoded in the functions  $p_1$  and  $p_2$ , which are in general functions of  $t$ ,  $x_1$ ,  $x_2$ ,  $y_1$ ,  $y_2$  and, vice versa, given the rules of an autonomous control, the fractions  $p_1$ ,  $p_2$  can be calculated. For the Scenarios 2 and 3 the corresponding expressions can be found in [15].

Like the processing rates, so are their expressions different for different situations at the queues. Obviously, if both queues are non-empty at time  $t$ , then

$$p_1(t) = 1 \quad \text{and} \quad p_2(t) = 0$$

hold. For  $x_1(t)+y_1(t)=0$ ,  $x_2(t)+y_2(t)>0$  one can derive

$$p_1(t) = 1, \quad p_2(t) = \min\left(1, \frac{(b_{11} - a_1(t))_+ b_{12}}{b_{11} a_2(t)}\right).$$

The corresponding expressions in the other cases, also for the third scenario, can be found in [15]. We note that the autonomous control rules in these scenarios are assigned to the parts. One can also allow the machines to decide in which order to process the parts or how fast to process them. In the latter case the processing rates  $b_{ij}$  become functions of  $t, x_1, x_2, y_1, y_2$ .

These simple examples illustrate how autonomous control can be defined, how it enters the equations and how it affects the dynamic behaviour of a logistic network.

## Conclusions

We have classified possible models for autonomous logistic processes and discussed how an autonomous control enters these models and what its effects on the dynamics and stability of the processes are. An example illustrates the answers to these questions. We discussed the advantages of autonomous control and pointed out the related stability problem.

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